Computations of Unsteady Transonic Aerodynamics Using Prescribed Steady Pressures

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An inverse procedure is used to obtain the steady-state flowfield and the airfoil shape corresponding to an input steady pressure distribution. Unsteady effects can then be studied using the time-linearized code UTFC, or the LTRAN2 code using the airfoil shape found by the inverse procedure. Experimental results are used as input and the computed unsteady response is compared with the measured values. Very good agreement between the results demonstrates the importance of an accurate steady flowfield in predicting unsteady responses.

Nomenclature

C_p = pressure coefficient
f_{ij} = intermediate solution values
k = reduced frequency based on the chord
M_∞ = freestream Mach number
t = time in chord length travel
VL = scaled lower airfoil slopes
VU = scaled upper airfoil slopes
x,y = coordinates
$Y^0(x)$ = airfoil ordinates
α = acceleration parameter
γ = ratio of specific heats
δ = amplitude of unsteady motion of airfoil
$\vec{\delta}_x, \overleftarrow{\delta}_x$ = forward and backward difference operator in x direction
τ = airfoil thickness
ϕ = perturbed velocity potential
ϕ^0 = steady velocity potential
ϕ_{ij} = value of ϕ at node points i and j
ω = angular frequency

Introduction

THE failure of linear theory in predicting unsteady transonic responses, especially when shock waves are present, is well documented and discussed by many authors (see, e.g., Tijdeman and Seebass[1]). To achieve accurate unsteady predictions, not only must the motion of the shock be properly taken into account, but the shock location and strength about which the unsteady motions occur also must be correct. While the unsteady transonic small-disturbance theory[2] is quite capable of predicting small-amplitude unsteady motions, its steady counterpart often gives erroneous predictions due to the local failure of the governing equation at the leading edge and the implied limitation on shock strength. Although these problems can be corrected by using numerical codes based on the full potential equations,[3,4] or even better, the Euler[5] or Navier-Stokes equations,[6] the cost of using such codes in engineering applications is prohibitively high. When strong shock waves are present, one still must rely on experimental results although they may be contaminated by wind tunnel wall effects.

Here a simple inverse method is proposed that uses the transonic small-disturbance equation to establish a known steady-state flowfield which is then subjected to unsteady changes in the airfoil attitude or shape. The known steady pressure distribution on an airfoil is prescribed as input instead of the airfoil slopes. This steady pressure distribution can be obtained from wind tunnel measurements or numerical computations. A modified version of the AF2 scheme of Ballhaus et al.[7] IAF2, is used to find the airfoil slopes corresponding to the given steady pressure distribution. An arbitrary integration constant is added to the leading-edge potential to satisfy closure as well as to insure smoothness of the airfoil. For all cases tested this procedure converged to the desired accuracy. In particular, for input pressures from the original AF2 analysis code, it recovers the airfoil slopes accurate to the order of the truncation error in the input pressure distribution.

The inverse process involved here follows the same relaxation procedure as in AF2. Once a converged solution is reached, the corresponding airfoil shape is determined, and the unsteady flowfield then can be computed using either UTFC (a code developed based on time-linearized small-disturbance theory[8]) or LTRAN2.[2]

Governing Equations

The unsteady, two-dimensional, transonic small-disturbance equation written in dimensionless form is

$$-k^2M_\infty^2\phi_{tt} - 2kM_\infty^2\phi_{xt} + [1 - M_\infty^2 - (\gamma+1)M_\infty^2\phi_x]\phi_{xx} + \phi_{yy} = 0 \qquad (1)$$

and the boundary condition on the body takes the form

$$\phi_y(x,0,t) = \tau[Y_x^0 + \delta/\tau(Y_x^u + kY_t^u)], \quad 0 \le x \le 1 \qquad (2)$$

where the instantaneous body shape has been decomposed into a steady part Y^0 and an unsteady part Y^u; across the wake at $y=0$,

$$[C_p] = [\phi_x + k\phi_t] = 0, \qquad 1 < x \qquad (3)$$

i.e., the pressure coefficient C_p is continuous.

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At large distances from the airfoil, the boundary conditions prescribed in Ref. 10 are adopted to minimize reflections from the numerical boundary in the far field.

As Houwink and van der Vooren[9] have shown, significant improvements to the numerical results, when compared with experimental results, are obtained by retaining all but the first term of Eq. (1). Hence, these modifications are adopted here and the terms of $0(k)$ in Eqs. (2) and (3) are retained.

Numerical Algorithm

One of the best ways of solving the problem posed by Eqs. (1-3) is to first obtain the steady-state flowfield corresponding to a prescribed airfoil shape $\tau Y^0(x)$, or to a prescribed pressure distribution characterized by $\phi^0_x(x,0)$. The AF2 scheme of Ballhaus et al.[7] is suitable for the former. IAF2 is developed here for the latter case. The AF2 scheme, which approximates the steady part of Eq. (1), can be written for the subsonic region (see Ref. 7) as

$$(\alpha - A_i\overrightarrow{\delta}_x)f_{ij} = (\alpha^2\overleftarrow{\delta}_x - A_i\overrightarrow{\delta}_x\delta_{yy})\phi^n_{ij} \tag{4a}$$

and

$$(\alpha\overleftarrow{\delta}_x - \delta_{yy})\phi^{n+1}_{ij} = f_{ij} \tag{4b}$$

where $A_i = [1 - M^2_\infty - (\gamma+1)M^2_\infty\delta_x\phi^n_{ij}]$.

Equation (4a) can be solved alternately at every grid point, except for those on the airfoil. Because the steady pressure $-2\phi^0_x$ is given on the airfoil at all times, the second derivative ϕ_{yy}, can also be computed from Eq. (1); therefore, the values of f_{ij} are always known on the airfoil. Thus, Eq. (4a) can be solved for f_{ij} for each i from the first grid point to the leading edge and from the trailing edge to the last grid point. In the y sweep, the values of ϕ_{ij} on the airfoil are known up to an integration constant C. To update the values of ϕ_{ij} on the airfoil, (following Tranen[11]), let

$$\phi^{n+1}_{ij} = \phi^n_{ij} + C$$

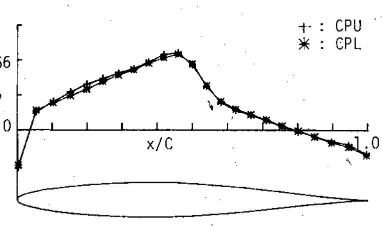


Fig. 1 Upper and lower steady surface pressure from the experiments of Ref. 13 for an NACA 64A010 airfoil, $M_\infty = 0.8$.

One way to choose C is to consider the mass flux across the airfoil

$$\frac{2C}{\Delta y} = \int_0^1 [\phi_y(x,0^+) - \phi_y(x,0^-)]\mathrm{d}x \tag{5}$$

where 0^+ and 0^- denote upper and lower surface of the airfoil and Δy (a fictitious time step) the distance of the first grid line from the airfoil. These fluxes are counterbalanced by an unsteady pressure distribution,

$$\delta_t\phi^n_{ij} \sim (\phi^{n+1}_{ij} - \phi^n_{ij})/\Delta y$$

on the airfoil. The constant C should approach zero or a small value as the solution reaches a steady state, indicating that a required degree of closure of the airfoil has been achieved. Note that the steady-state value of this constant is not important; it depends on the density of the grid as well as the way in which the integral of Eq. (5) is computed.

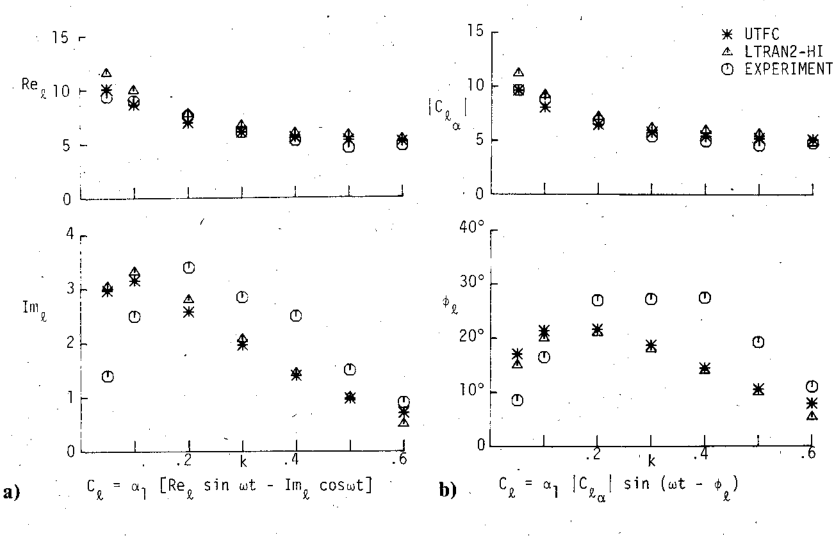
Given the same grid, IAF2 recovers the airfoil input to AF2 accurately with the corresponding pressure distribution from AF2. For all pressure distributions tested (see Chung[12] for details), IAF2 yields a steady-state flowfield and a smooth body slope distribution with only a minor adjustment of the steady-state constant C [which may be nonzero since only $\phi_x(x,0^+)$ is given]. The object is to compute a steady flowfield for subsequent unsteady computations, and not, in principle, to recover a specific airfoil; nevertheless, the implementation of this procedure in the design of airfoils with a given pressure distribution is routine. However, that is not the thrust of the research reported here.

Once the steady flowfield and the corresponding airfoil have been obtained, the theory described in Ref. 8 can be used to determine the unsteady response for a given mode of motion for the body with the prescribed steady pressure distribution. LTRAN2 of Ref. 2 may be used for unsteady motions with moderate amplitudes.

Results and Discussion

Because of the efforts made to reduce wind tunnel wall effects, the experimental data of Davis and Malcolm[13] are chosen to demonstrate the merits of the present method of determining unsteady transonic responses. Figure 1 shows the experimental pressure distribution for an NACA 64A010 airfoil at $M_\infty = 0.8$, except at the leading edge and the trailing edge where the values have been calculated using IAF2. Because there are only 19 measured pressure values from these experiments, a grid of 23 mesh points on the airfoil was chosen for all computations. The full grid used is 100×80; a smaller grid would be satisfactory. The input pressure values are first integrated and the resulting potential interpolated for the values at the mesh points. The equations are then solved

Fig. 2 Lift coefficients vs reduced frequency for a pitching NACA 64A010 airfoil, $M_\infty = 0.8$. a) Real and imaginary components. b) Amplitude and phase.



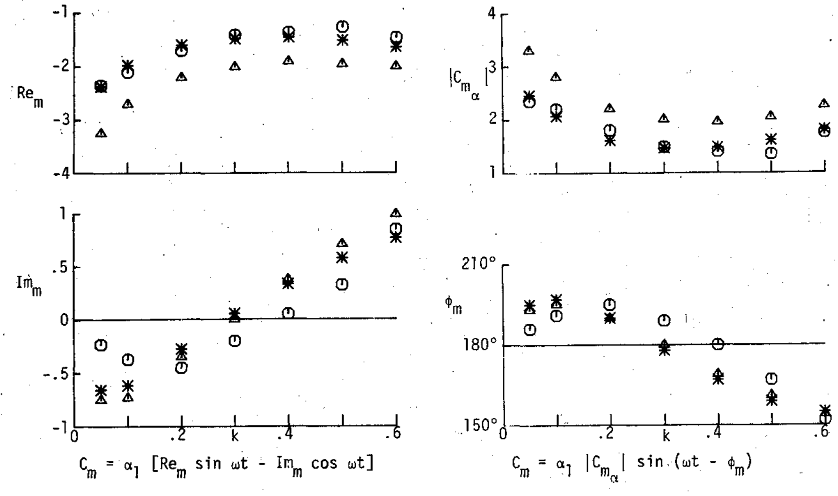


Fig. 3 Leading-edge moment coefficients vs reduced frequency for a pitching NACA 64A010 airfoil, $M_\infty = 0.8$. a) Real and imaginary components. b) Amplitude and phase.

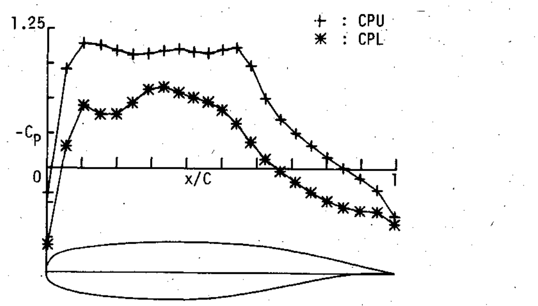


Fig. 4 Upper and lower steady surface pressure from the experiments of Ref. 13 for the supercritical wing section NLR-7301; $M_\infty = 0.752$, mean angle of attack 0.37 deg.

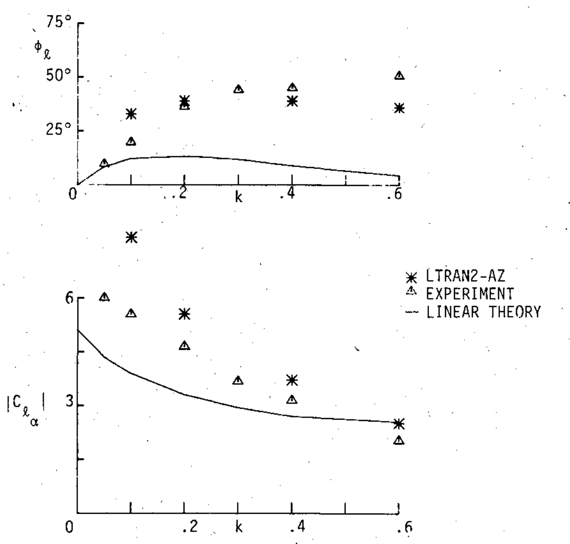


Fig. 5 Amplitude and phase of the lift coefficients vs reduced frequency for a pitching NLR-7301 airfoil about 0.4 chord; $M_\infty = 0.752$, mean angle of attack 0.37 deg.

using IAF2 to the order of accuracy normally required for direct analysis; that is, several orders-of-magnitude reduction in the maximum residue, and a stable supersonic zone defined by the number of supersonic points counted. Unchanging airfoil slopes are required for this inverse problem. Normally 500 iterations are sufficient for convergence.

The UTFC code is used to compute the unsteady aerodynamic responses using the steady flowfield computed by IAF2 as input. Figures 2 and 3 show good agreement with the experimental measurements. The magnitudes of the lift

Table 1 Scaled upper and lower body slopes obtained using IAF2 for the input pressure distribution in Fig. 4

I	*X*	*VU*	*VL*
41	0.00308	4.21726	−2.88857
42	0.05371	1.81367	−2.42406
43	0.10310	0.92771	−1.04363
44	0.15129	0.55063	−0.68533
45	0.19835	0.35350	−0.56141
46	0.24433	0.27050	−0.42987
47	0.28928	0.14408	−0.19654
48	0.33324	−0.01957	0.03921
49	0.37625	−0.15283	0.21993
50	0.41836	−0.25812	0.38493
51	0.45959	−0.26356	0.57244
52	0.50000	−0.41159	0.78874
53	0.54041	−0.56173	1.02741
54	0.58164	−0.75542	1.21431
55	0.62375	−0.79845	1.24433
56	0.66676	−1.16106	1.21801
57	0.71072	−1.28019	1.20038
58	0.75567	−1.36161	1.13375
59	0.80165	−1.39721	1.00465
60	0.84871	−1.40179	0.83672
61	0.89690	−1.40804	0.65448
62	0.94629	−1.41480	0.51379
63	0.99692	−1.53449	0.59681

and moment coefficients are recovered almost exactly and the phase angles are also in close agreement with the results of Hessenius and Goorjian[14] using LTRAN2-HI (an updated version of LTRAN2 that includes the high-frequency modification mentioned earlier). The fact that the agreement between the experimental and computed phase angles is only moderately good may be due either to viscous effects or, much more likely, to contamination of the experimental phase lags by waves reflected from the wind tunnel walls. This effect has been noted in an earlier investigation.[10]

It appears that UTFC, a time-linearized version of LTRAN2-HI, achieves better agreement with the experiment than its full nonlinear version. This may be attributed to the fact that in the full-linearized theory, shocks are treated as moving discontinuities, while in LTRAN2 they are smeared by artificial viscosity over a few grid points.

To test the method over more severe flow conditions, the experimental pressure distribution shown in Fig. 4 for the supercritical wing section NLR-7301 at almost shock-free conditions was used. The slopes (scaled by the thickness of the original airfoil), obtained from IAF2, of the upper and lower surface, are shown in Table 1. A direct analysis using the AF2 code with these slopes as input shows almost no change in the pressure distribution. A modified version of LTRAN2, LTRAN2-AZ was used because the amplitudes of motion (0.5 deg) lead to nonlinear unsteady behavior for this nearly shock-free flow. Unsteady responses to 0.5 deg pitching motions about 40% of the chord for the frequencies shown by the stars in Fig. 5 were then computed. For the reduced frequency of 0.1, 0.25 deg pitch amplitude was used in order to keep the shock on the airfoil. Eight cycles of harmonic motion were computed for each reduced frequency with a time step of 3 deg. The last cycle was used for comparison with the experiment. Again, good agreement in magnitude and in phase angle are obtained. The experimental data obtained from Ref. 15 and the present results are for the upper surface only, and the nonzero mean values of the harmonic lift coefficient have been subtracted in the comparison.

Conclusion

An inverse method for computing the steady flowfield and the body slope that generates it has been developed, from which an accurate prediction of the unsteady response of a transonic airfoil can be obtained efficiently. It is implied that an accurate steady state with correct shock jump and shock location is vital to the prediction of unsteady responses. The experimental savings in using this procedure to determine flutter boundaries are substantial and warrant its application in three-dimensional studies.

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